# CS 369: Introduction to Robotics

Prof. Thao Nguyen

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HAVERFORD
COLLEGE

# Outline for today

- Coordinate frame transformations
- Robot kinematics

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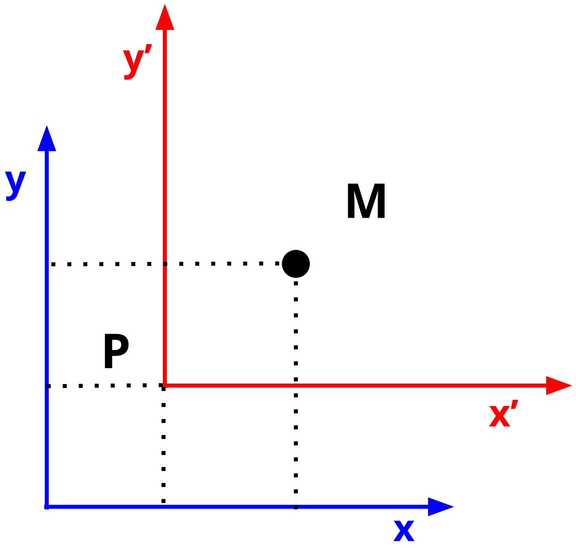
- Robot kinematics

# Robot pose

- Refers to the robot’s position and orientation in the environment
  - of which part of the robot?
  - with respect to which reference?

- Reference frame: static coordinate system

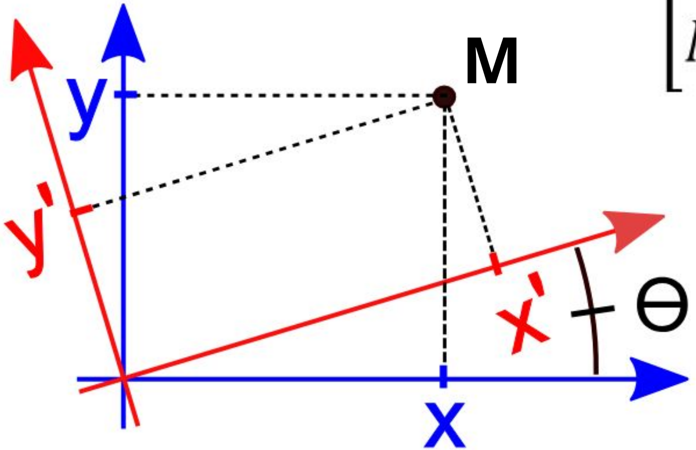
# Translation




$$\begin{bmatrix} M_x \\ M_y \end{bmatrix} = \begin{bmatrix} P_x \\ P_y \end{bmatrix} + \begin{bmatrix} M_{x'} \\ M_{y'} \end{bmatrix}$$

p (position vector)

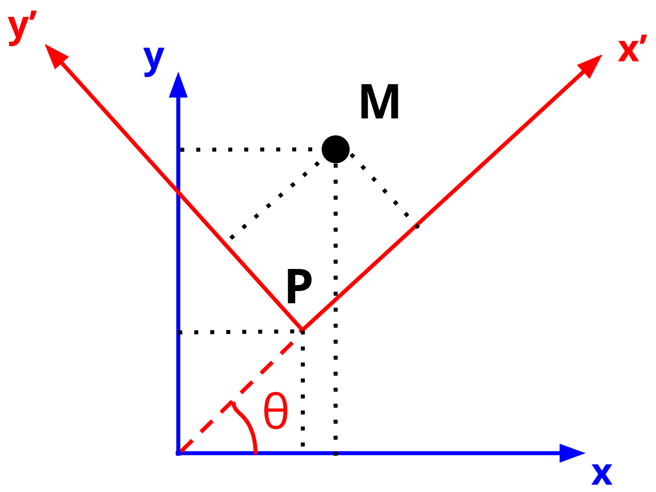
# Rotation



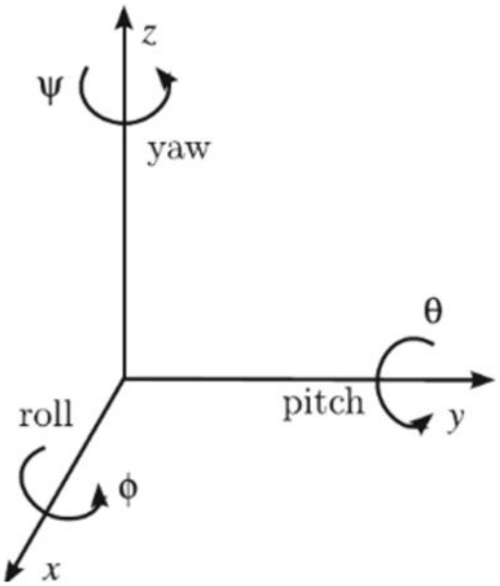

$$\begin{bmatrix} M_x \\ M_y \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}}_{R(\theta)} \begin{bmatrix} M_{x'} \\ M_{y'} \end{bmatrix}$$

# Homogeneous transformation matrix

- Combines translation and rotation into a single matrix multiplication
- Standard form: $T = \begin{bmatrix} R & p \\ 0^T & 1 \end{bmatrix}$
- $$\begin{bmatrix} M_x \\ M_y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & P_x \\ \sin(\theta) & \cos(\theta) & P_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} M_{x'} \\ M_{y'} \\ 1 \end{bmatrix}$$



# 3D transformations




$$p = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} \qquad R = R_x R_y R_z$$

$$\mathbf{R}_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{pmatrix}$$

$$\mathbf{R}_y = \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{pmatrix}$$

$$\mathbf{R}_z = \begin{pmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Rotations in 3D

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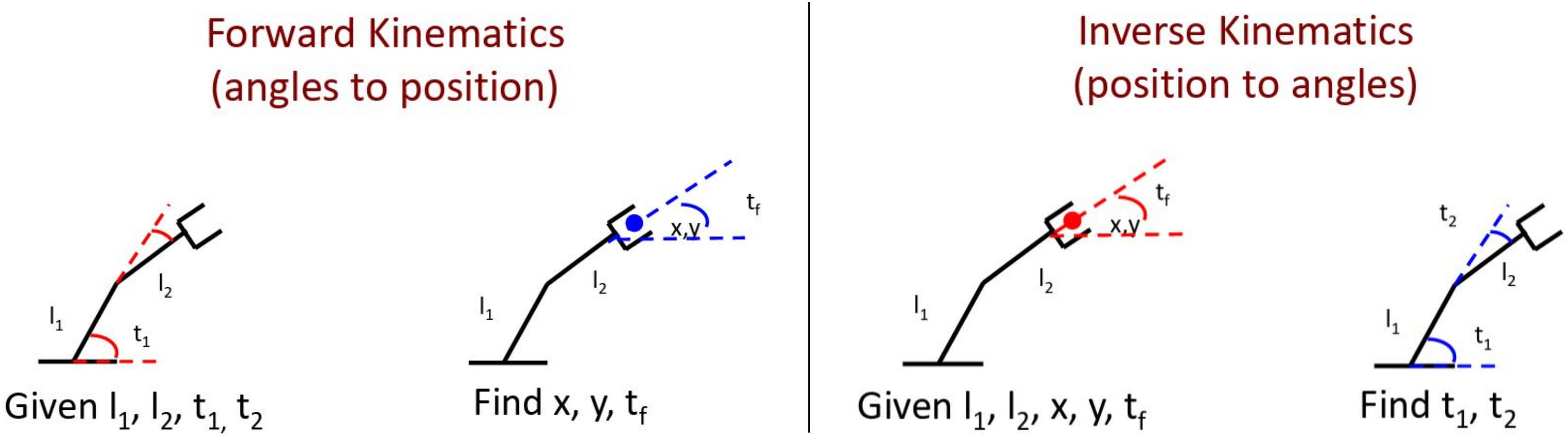
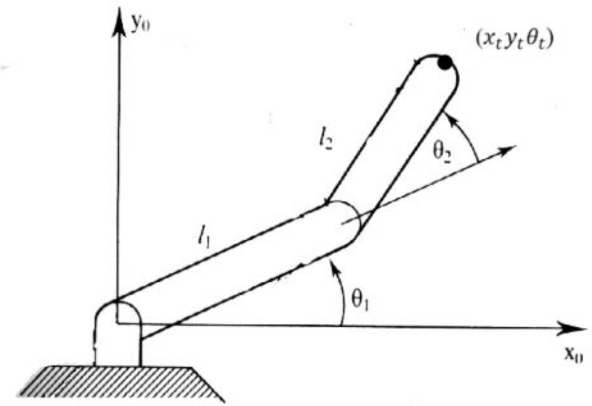
# Robot kinematics

Refers to the geometry and movement of robotic mechanisms



# Forward kinematics



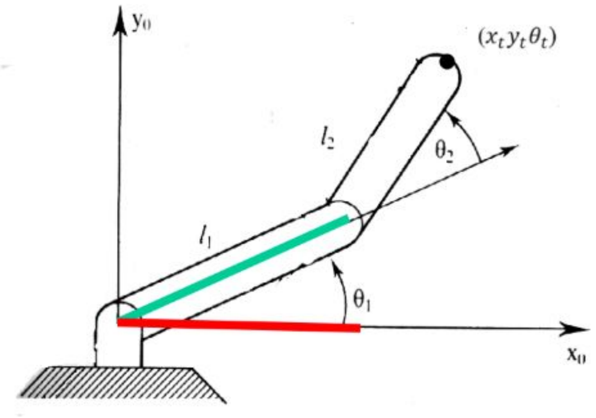

**Set up:**

- You have an RR robotic arm with base at the origin.
- The first link moves th1 with respect to the x-axis. The second link moves th2 with respect to the first link.

**Question:**

- What is the position and orientation of the end effector of the robotic arm?

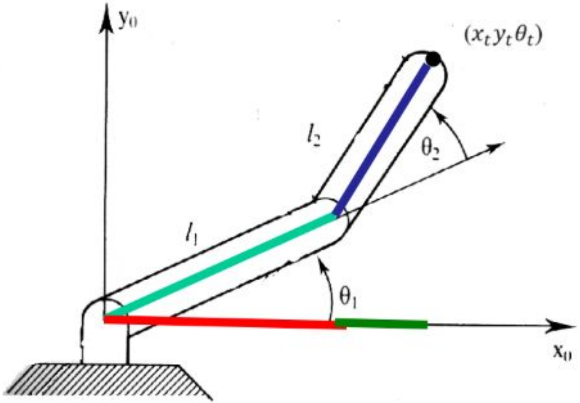
# Geometric approach




$$\theta t = \theta 1 + \theta 2$$

$$x_t = l1 * \cos(\theta 1)$$

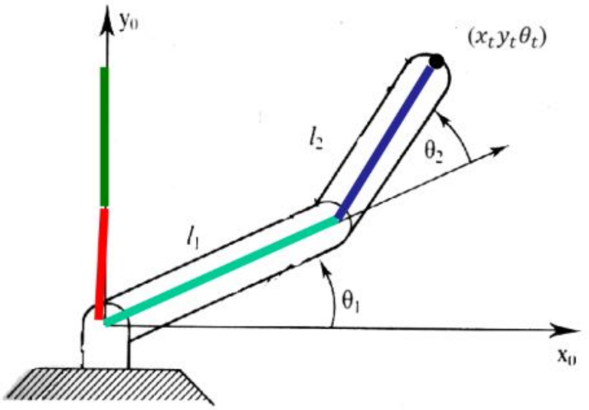
# Geometric approach




$$\theta t = \theta 1 + \theta 2$$

$$x_t = l1 * \cos(\theta 1) + l2 * \cos(\theta 1 + \theta 2)$$

# Geometric approach

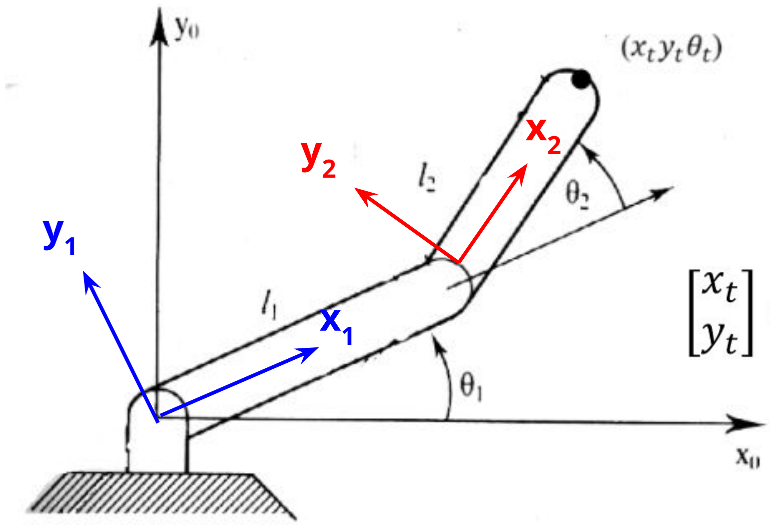



$$\theta t = \theta 1 + \theta 2$$

$$x_t = l1 * \cos(\theta 1) + l2 * \cos(\theta 1 + \theta 2)$$

$$y_t = l1 * \sin(\theta 1) + l2 * \sin(\theta 1 + \theta 2)$$

# Algebraic approach




$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) \\ \sin(\theta_1) & \cos(\theta_1) \end{bmatrix} \left( \begin{bmatrix} l_1 \\ 0 \end{bmatrix} + \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) \end{bmatrix} \begin{bmatrix} l_2 \\ 0 \end{bmatrix} \right)$$